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Stability of neutrinos in the singlet majoron model

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Abstract

We show that there is no one-loop enhancement of the rate for a light neutrino to decay into a lighter neutrino plus a majoron, contrary to a recent claim. Thus the light neutrinos must satisfy the cosmological bound of having masses less than 35 eV in the singlet majoron model, or else violate the constraint imposed by galaxy formation. In the latter case, ν_τ could have a mass between 40 keV and 1 MeV, while satisfying all other cosmological constraints.

The singlet majoron model [1] is one of the simplest imaginable extensions of the standard model of particle physics. It consists of adding one or more right-handed neutrinos, and a complex scalar field σ with lepton number -2 . If σ has a large vacuum expectation value v_σ , one has a natural explanation for the smallness of neutrino masses through the seesaw mechanism [2]. In addition there is a massless Goldstone boson χ , the majoron, which couples primarily to neutrinos with a strength suppressed by powers of v_σ . Although the majoron would be impossible to see directly, it could play a role in cosmological and astrophysical settings.

One of the potential uses for majorons is in evading bounds on neutrino masses. For example a tau neutrino mass between 35 eV and 1 MeV would seriously conflict with present estimates of the energy density of the universe since the standard model decay $\nu_\tau \rightarrow \nu_\mu \gamma$ is too slow to deplete the population of such neutrinos even by the present time. The rate for decay into a majoron rather than a photon can be many orders of magnitude faster, thus alleviating the conflict.

In nonminimal majoron models with several scalar fields, it is certainly true that neutrino decay rates become unsuppressed [3]. But with only one scalar field, it turns out that the matrix of couplings of the majoron between the various light neutrinos is proportional to the mass matrix, at lowest order in the small parameter $\epsilon \sim m_{\text{Dirac}}/v_\sigma$ [4]. The transition $\nu \rightarrow \nu' \chi$ only occurs at higher order in the small parameter, which typically makes it as irrelevant as the electromagnetic decay. It is therefore an interesting claim [5, 6] that the one-loop correction to the amplitude for $\nu \rightarrow \nu' \chi$ is nonzero at leading order in ϵ and is diminished only by typical loop suppression factors. This claim has already been used in the construction of at least one model [7].

Here we wish to demonstrate the incorrectness of the claim that loop effects enhance the neutrino decay rate. We have done this in two different ways. The first method is to compute the one-loop effective Lagrangian for the light neutrinos. Let ν denote the vector of three light neutrino states, and let μ and g denote the 3×3 tree-level mass and majoron-coupling matrices respectively. Taking into account one-loop contributions, denoted by $\delta\mu$ and δg ,

the effective Lagrangian becomes

$$-\frac{1}{2}\bar{\nu}(\mu + \delta\mu)\nu - \frac{i}{2}\chi\bar{\nu}\gamma_5(g + \delta g)\nu, \quad (1)$$

We will show that $g + \delta g$ is proportional to $\mu + \delta\mu$ at leading order in ϵ , just like the tree-level quantities, so that there is no loop enhancement of the decay rate.

In the effective Lagrangian approach, only one-particle irreducible (1PI) diagrams are included. One must then rotate the full matrix of couplings $g + \delta g$ to the basis where the mass matrix $\mu + \delta\mu$ is diagonal to obtain the physical decay amplitudes. It is also possible to calculate the amplitude directly, including the one-particle reducible (1PR) self-energy corrections to the tree level vertex, as in reference [6]. An example of such a contribution is shown in Fig. 1. We have repeated this computation and found discrepancies with ref. [6]. (We note that it is crucial to put the internal neutrino ν_2 in the 1PR diagram on the mass shell of the external neutrino ν_1 rather than at zero momentum, in order to get the correct result.) In this approach, the 1PR diagrams perform the function of rotating to the one-loop mass eigenbasis. We have verified that the 1PR and 1PI diagrams cancel each other, giving a vanishing decay amplitude at lowest order in ϵ . Because the effective Lagrangian approach is simpler, we will present our results only for that method.

Before giving the explicit calculations we observe that our result is not a mere accident, but can be shown by a general argument. Consider the tree-level Lagrangian for the singlet majoron model,

$$-h_{ij}\bar{L}_i H P_R \nu_{R,j} - f_{ij}\sigma\bar{\nu}_{R,i} P_R \nu_{R,j} + \text{h.c.} - \lambda\left(|\sigma|^2 - v_\sigma^2/2\right)^2, \quad (2)$$

where H, L are the Higgs and lepton doublets, $P_R = (1 + \gamma_5)/2$ and $\nu_{R,i}$ are the isosinglet right-handed neutrinos (written in Majorana form). Now imagine doing the renormalizations up to any desired order in the *symmetric* phase, *i.e.*, for negative v_σ^2 . The mass terms remain zero because they are protected by the lepton number and gauge symmetries and only the Yukawa couplings get renormalized. The renormalized couplings should have an analytic dependence on v_σ^2 , so that we can compute one-loop effects in the broken phase simply by substituting a positive value for v_σ^2 into the above results. Then the proportionality of

resulting renormalized masses and couplings is manifest just as for the tree level parameters, to *all* orders in the loop expansion. While this argument alone might suffice to prove our point, we feel obliged to show explicitly how the correct result can be derived in the broken phase, where the computation is more involved.

The first step is to rewrite the Lagrangian when the Higgs and σ fields have acquired their VEV's. There is a Dirac mass matrix $m = hv/\sqrt{2}$ connecting the left- and right-handed neutrinos, where $v = 246$ GeV is the VEV of the Higgs, and a Majorana mass matrix $M = \sqrt{2}fv_\sigma$ for the right-handed neutrinos alone. Assuming the eigenvalues of M are much larger than the entries of m , the full mass matrix can be partially diagonalized by

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = \begin{pmatrix} 1 & \epsilon \\ -\epsilon^T & 1 \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}; \quad \epsilon \equiv mM^{-1}, \quad (3)$$

so that ν and N respectively have the light and heavy mass matrices $\mu = -mM^{-1}m^T$ and M , to leading order in ϵ . We also expand

$$\sigma = \frac{1}{\sqrt{2}}(v_\sigma + \rho + i\chi). \quad (4)$$

In contrast to the massless majoron χ , ρ has a mass given by $m_\rho^2 = 2\lambda v_\sigma^2$. The interaction Lagrangian is then

$$\mathcal{L}_i = -\frac{1}{2v_\sigma} \begin{pmatrix} \bar{\nu} & \bar{N} \end{pmatrix} \begin{pmatrix} \epsilon M \epsilon^T & -\epsilon M \\ -M \epsilon^T & M \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix} - \frac{\lambda}{4}(2v_\sigma \rho + \rho^2 + \chi^2)^2, \quad (5)$$

from which the Feynman rules are easily deduced. The above-emphasized proportionality of the $\nu\nu\chi$ coupling matrix to the light mass matrix μ is manifest:

$$g = \frac{\epsilon M \epsilon^T}{v_\sigma} = -\frac{\mu}{v_\sigma}. \quad (6)$$

The next step is to examine the general structure of the one-loop shifts in the mass and coupling matrices. In the complete space of the light and heavy neutrinos, (ν, N) , the masses have the form

$$\text{masses} = \frac{1}{2} \begin{pmatrix} \mu + \delta\mu & \delta m \\ \delta m^T & M + \delta M \end{pmatrix}. \quad (7)$$

which necessitates re-diagonalizing the light and heavy states using the transformation

$$O = \begin{pmatrix} 1 & \delta\epsilon \\ -\delta\epsilon^T & 1 \end{pmatrix}; \quad \delta\epsilon = \delta m M^{-1}. \quad (8)$$

Here and below we have kept only the terms which are of leading order in the combined ϵ and loop expansion. (For example, keeping δM in the definition of $\delta\epsilon$ would be consistent only if we were doing a two-loop calculation.) After the rotation (8), $\mu + \delta\mu$ gets a further shift $-\delta m^T M^{-1} \delta m$ which is equivalent to a two-loop effect and can be ignored. But there is a nonnegligible shift in the matrix of couplings, which upon re-diagonalization undergoes

$$\text{couplings} = \frac{1}{2} \begin{pmatrix} g + \delta g & g' + \delta g' \\ g'^T + \delta g'^T & G + \delta G \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} g + \delta g - g' \delta\epsilon^T - \delta\epsilon g'^T & * \\ * & * \end{pmatrix}, \quad (9)$$

where $*$ indicates the terms we are not interested in. Thus what we called δg in eq. (1) is actually given by the naive one-loop shift plus terms due to heavy-light mixing at one loop. *We will henceforth use δg to denote the ‘naive’ contribution.* From this analysis we see that it is necessary only to compute the quantities δg , δm , and $\delta\mu$. These are given, respectively, by the Feynman diagrams shown in figures 2, 3 and 4.

First consider δg , figs. 2a-b. Although there are additional digrams not shown, with ν rather than N running in the loop, they are of higher order in ϵ . Adding the contributions of fig. 2 evaluated at vanishing external momenta, we obtain the matrix equation

$$\delta g = m_\rho^2 v_\sigma^{-3} \epsilon I(M, m_\rho) \epsilon^T, \quad (10)$$

with

$$I(x, y) = \frac{1}{16\pi^2} \frac{x^3}{x^2 - y^2} \ln \frac{x^2}{y^2}. \quad (11)$$

The factor of m_ρ^2 in the numerator here and below always comes from the difference between the propagators of χ and ρ , or from the fact that the $\chi\chi\rho$ coupling is proportional to m_ρ^2 . Counting powers of v_σ (remember that $m_\rho \sim M \sim v_\sigma$), we see that δg is of order $\epsilon^2 \sim (m/M)^2$, and is not proportional to the tree level mass matrix μ . If this were the end of the story, δg would cause fast decays among the light neutrinos. But we must also consider the mass shifts.

The Dirac mass shift δm is given by fig. 3. A straightforward calculation shows that

$$\delta m = -m_\rho^2 v_\sigma^{-2} I(M, m_\rho) \epsilon^T. \quad (12)$$

Using $\delta\epsilon = \delta m M^{-1}$ from eq. (8) and $g' = -\epsilon M/v_\sigma$ from eq. (5), we find that

$$\delta g_{\text{TOT}} = \delta g - g' \delta\epsilon^T - \delta\epsilon g'^T = -\delta g, \quad (13)$$

so the net effect of rotating away the induced heavy-light mixing is to reverse the sign of the naive shift in the couplings.

The final singlet Higgs contribution is the direct shift in the Majorana mass matrix of the light neutrinos, fig. 4, which we find to be given by $\delta\mu = v_\sigma \delta g$. Therefore the relation between the coupling and mass matrices at one loop and leading order in ϵ is

$$g + \delta g_{\text{TOT}} = -\frac{\mu + \delta\mu}{v_\sigma}, \quad (14)$$

that is, they are exactly proportional to each other. Thus they are simultaneously diagonalized and there is no loop-enhancement of decays.

From the general argument given above, it should be clear that our results do not depend on what kind of interactions are involved, so long as they do not explicitly break the global lepton number symmetry. Thus the one-loop effects involving Higgs and weak gauge bosons must also preserve the proportionality between masses and couplings. We have also carried out this calculation.

The relevant diagrams are shown in figs. 5 and 6 for δg and $\delta\mu$ respectively. In contrast to the previous case, simple power counting (taking into account necessary insertions of neutrino masses) shows that the electroweak contribution to δm is $O(\epsilon^3)$ and so can be neglected. The Z -boson interactions may be written as

$$\frac{-g}{4 \cos \theta_w} \begin{pmatrix} \bar{\nu} & \bar{N} \end{pmatrix} \begin{pmatrix} 1 & \epsilon \\ \epsilon^T & \epsilon^T \epsilon \end{pmatrix} \gamma_\mu \gamma_5 \begin{pmatrix} \nu \\ N \end{pmatrix} Z^\mu, \quad (15)$$

using the fact that the vector current vanishes for Majorana fields. In evaluating the diagrams it proves convenient to work in R_ξ gauge with $\xi = M_H^2/M_Z^2$, for then the contributions from

the real and imaginary part of the Higgs field exactly cancel each other, and we are left with only the gauge boson diagrams. We obtain

$$\delta\mu = -\frac{g^2}{4\cos^2\theta_w}\epsilon\left(3I(M, M_Z) + (M_H^2/M_Z^2)I(M, M_H)\right)\epsilon^T, \quad (16)$$

where I is given in eq. (11); notice that the relative contributions of the Higgs boson and the three polarizations of the Z are evident. We also find that

$$\delta g_{\text{TOT}} = \delta g = -\frac{\delta\mu}{v_\sigma}, \quad (17)$$

explicitly showing that the weak interactions also preserve the proportionality between masses and couplings that forbids transitions between light neutrinos at $O(\epsilon^2)$ in the amplitude.

(While the contribution of fig. 7 might at first appear to be relevant, it is $O(\epsilon^4)$. The subgraph containing Z_μ - χ mixing is of order $\epsilon^2 q_\mu$, where q_μ is the majoron momentum. Integrating by parts the effective operator $\bar{\nu}\gamma_\mu\nu\partial_\mu\chi$ and using the equations of motion gives another factor of the light neutrino mass, $\mu \sim \epsilon^2$.)

Finally, we examine the tree level decay rate more explicitly than appears to have been done elsewhere. With a little effort, one can go to next order in ϵ in the diagonalization of the neutrino mass matrix, to find that the off-diagonal block of the heavy-light rotation matrix, ϵ in eq. (3), should be supplemented by the term $-mM^{-1}(m^T m M^{-1} + \frac{1}{2}M^{-1}m^T m)M^{-1}$. This is enough to determine the masses and majoron couplings for the light neutrinos to order ϵ^4 ,

$$\mu = \mu_0 - \frac{1}{2}\{\mu_0, \epsilon\epsilon^T\}, \quad g = -\frac{\mu}{v_\sigma} + \left\{\frac{\mu}{v_\sigma}, \epsilon\epsilon^T\right\} \quad (18)$$

where $\mu_0 = -\epsilon M \epsilon^T$. To more easily explore the consequences of this result, let us assume there is mixing only among the last two generations, with eigenvalues m_{ν_i} and M_i for the light and heavy masses. In the mass basis, one can show that the off-diagonal coupling in eq. (18) is given by

$$g_{23} = \frac{1}{2}\sin 2\alpha\left(M_2^{-1} - M_3^{-1}\right)(m_{\nu_2}m_{\nu_3})^{1/2}(m_{\nu_2} + m_{\nu_3})/v_\sigma, \quad (19)$$

where α is the rotation angle for diagonalizing the matrix $M^{-1/2}m^TmM^{-1/2}$. (Note that this vanishes if the heavy neutrinos are degenerate, as expected, since if M is proportional to the unit matrix, the Dirac masses can be diagonalized simultaneously with it, in which case there is no mixing or decay.) For the purpose of obtaining cosmological upper bounds on v_σ , we conservatively assume that the angle is large and the eigenvalues M_i have a large splitting, so that $g_{23} \sim m_{\nu_2}^{1/2}m_{\nu_3}^{3/2}/v_\sigma^2$, leading to a decay rate of

$$\frac{1}{\tau} \sim \frac{m_{\nu_2}m_{\nu_3}^4}{16\pi v_\sigma^4} \quad (20)$$

for the process $\nu_3 \rightarrow \nu_2\chi$.

If m_{ν_3} exceeds the cosmological upper bound of 35 eV for stable neutrinos [8], ν_3 must decay fast enough so that its relativistic decay products have time for their energy to be redshifted away. From eq. (20) it is clear that we want m_{ν_2} to be as large as possible. Demanding that the present energy density of ν_2 particles plus majorons from the decaying ν_3 not exceed the closure value, we obtain a bound involving the decay temperature T_D ,

$$m_{\nu_2} + \left(m_{\nu_2}^2 + p^2\right)^{1/2} + p < 35 \text{ eV} ; \quad p = \frac{1}{2} \frac{m_{\nu_3}T_0}{T_D}, \quad (21)$$

where p is the redshifted momentum of the majoron. With the aid of eq. (20) and the time-temperature relationship for a universe dominated by nonrelativistic ν_3 's, we get

$$v_\sigma < 500 \text{ GeV} \left(\frac{m_{\nu_3}}{1 \text{ MeV}} \right)^{1/2}, \quad (22)$$

assuming m_{ν_2} has the optimal value (~ 9 eV) for giving the least restrictive bound.

If m_{ν_3} was smaller than 240 keV, this would imply that v_σ is below the weak scale of 246 GeV, a somewhat small value from the perspective of the seesaw mechanism for the neutrino masses. Nevertheless with $v_\sigma = 100$ GeV, the Dirac neutrino masses corresponding to $m_{\nu_1} \sim 1$ eV, $m_{\nu_2} \sim 10$ eV, $m_{\nu_3} \sim 40$ keV are 0.3 MeV, 1 MeV and 60 MeV, respectively, comparable to the smallest lepton and quark masses. If m_{ν_3} saturated the nucleosynthesis bound of 500 keV [9] then v_σ could be as large as 350 GeV and the neutrino Dirac masses could span a slightly greater range, up to 0.4 GeV for the third generation.

If the majoron has a gravitationally induced mass [10], whose natural value is around 1 keV, the above results still hold, except for the fact that now the final density of ν_2 will be four times its thermal value (due to the decay products of $\chi \rightarrow \nu_\mu \nu_\mu$), so that its mass should be somewhat smaller. The ensuing bound on v_σ decreases only slightly.

Although considerations of the cosmological energy density leave room for the relevance of light neutrino decays in the singlet majoron model, the growth of large scale structure leads to a different conclusion. The decay products of a massive tau neutrino could cause a second radiation-dominated era, during which density perturbations grow only logarithmically with time. The recent measurements of COBE [11] indicate that any additional period of radiation domination would conflict with the necessity for primordial perturbations to have grown into galaxies by now. Therefore ν_τ should never have matter-dominated the energy density of the universe [12], hence the decay temperature of ν_τ must not be much less than its mass (more precisely, $m_{\nu_3}/T_D < 14$; see ref. [13]), leading to the constraint

$$v_\sigma < 20 \text{ GeV} \left(\frac{m_{\nu_3}}{1 \text{ MeV}} \right)^{1/2} \left(\frac{m_{\nu_2}}{35 \text{ eV}} \right)^{1/4}. \quad (23)$$

For any allowed value of m_{ν_3} , this is too small a value of v_σ to give plausible seesaw masses for the light neutrinos.

In summary, we have given both a simple general argument valid to all orders in perturbation theory, and detailed calculations at one loop in the broken phase, to show that decay amplitudes between light neutrinos always vanish at order ϵ^2 in the simplest majoron model, where ϵ is the ratio between the scales of Dirac and Majorana masses of the neutrinos. Thus there is no enhancement of the amplitude $\nu \rightarrow \nu' \chi$ due to loop corrections in the simplest majoron model. We have also shown that the tree level decays are too slow to be cosmologically relevant, but only if the galaxy formation constraint is considered.

Acknowledgements

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Figure Captions

Fig. 1 One-particle reducible contribution to the decay amplitude.

Fig. 2a,b Singlet Higgs contributions to δg .

Fig. 3 Singlet Higgs contribution to δm .

Fig. 4 Singlet Higgs contribution to $\delta\mu$.

Fig. 5a,b Electroweak contributions to δg , using all four combinations of ν and N in 5b.

Fig. 6a,b Electroweak contributions to $\delta\mu$.

Fig. 7 A subleading contribution to δg .

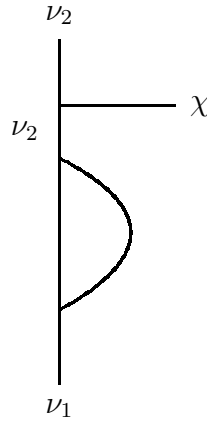


Fig. 1

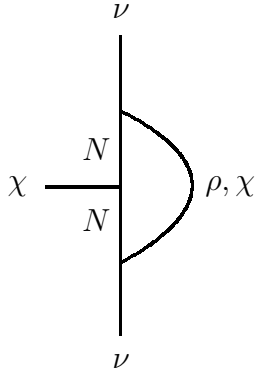


Fig. 2a

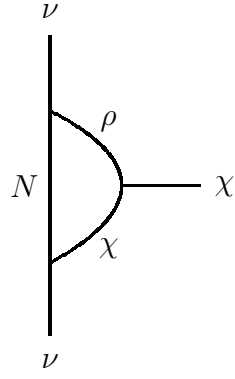


Fig. 2b

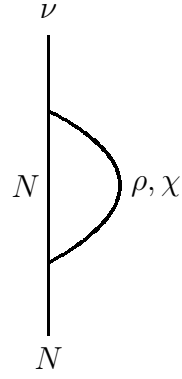


Fig. 3

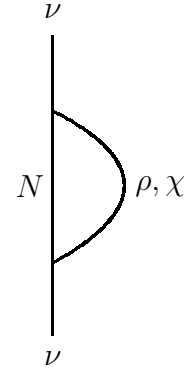


Fig. 4

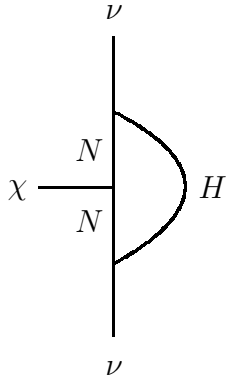


Fig. 5a

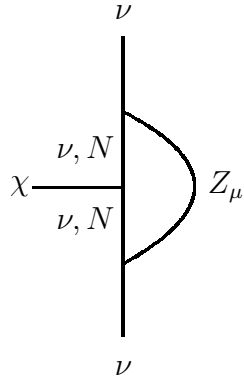


Fig. 5b

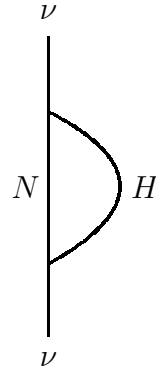


Fig. 6a

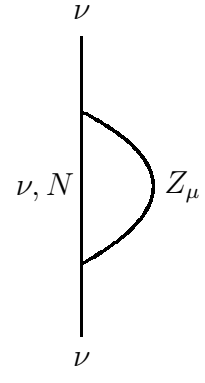


Fig. 6b

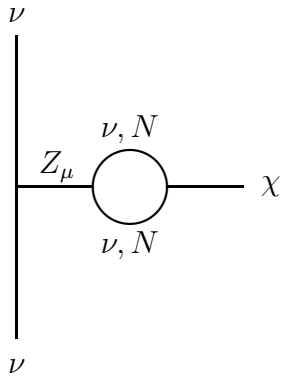


Fig. 7